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INFLUENCE OF SAFFMAN'S LIFT FORCE ON THE MOTION OF A PARTICLE IN A COUETTE LAYER

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The article is concerned with the study of the effect of E. S. Asmolov's corrections to Saffman's lift force for the wall vicinity and a nonzero ratio of Reynolds numbers. It is shown in what way these corrections change the particle paths in a Couette layer and the conditions of deposition.

A free-rotating particle moving in a shear flow gives rise to a transverse (lift) force. In [1] Saffman suggests the following formula for the linear profile of gas velocity:

$$P_S = 0.25C\rho_g\delta^2 (U_g - U_p) (\nu \partial U_g/\partial Y)^{1/2} . \tag{1}$$

At small Reynolds numbers, $\mathrm{Re}_\nu \ll 1$, $\mathrm{Re}_k \ll 1$ and at their ratio $A \equiv \mathrm{Re}_\nu/\mathrm{Re}_k^{1/2} \ll 1$, $C = 6.46 = \mathrm{const}$.

E. S. Asmolov [2, 3] considers the more-general case of an arbitrary value of A as well as the influence of the wall vicinity on the coefficient C. In the general case $C = C(A, \eta)$. In [3] an approximation of the coefficient C for the case $\eta \to \infty$: $C(A, \infty) = 6.46 f_1(A)$ was obtained:

$$f_1 (A) = 1/(1 + 0.581A^2 - 0.439A^3 + 0.203A^4) , \tag{2}$$

i.e., at $A = 0$ far from the wall $C = 6.46$.

The dependence of the coefficient C on the dimensionless distance to the wall is more complex; in [2] it is presented only in graphical form, thus making its use in calculations difficult. We introduce the function $f_2(A, \eta) = C(A, \eta)/C(A, \infty)$. According to the results of [2, 3], it can be approximated in a first approximation by the following exponential function:

$$f_2 (A , \eta) = 1 - \exp (- k (\eta - \eta_0)) . \tag{3}$$

Relation (3) accounts for the fact that the lift force reverses its direction when $\eta < \eta_0$. In this case, both η_0 and k depend on the value of A. According to the results of [2], in a first approximation for $0 \le A \le 2$ these dependences can be approximated by the curves

$$\eta_0 = 0.60A^{1/2} , \quad k = 0.439 + 0.093A^2 + 0.047A^3 . \tag{4}$$

The function $C = 6.46 f_1 f_2$ derived in this way was used in calculations.

Since the Reynolds numbers Re_ν are rather small, we can disregard the rotation of a particle and the Magnus lift force exerted on it [4] and use the resistance force in Stokesian form: $F_\mu = 3\pi\delta\nu\rho_g(V_g - V_p)$.

We will consider a Couette laminar gas layer in which the x axis is directed along a solid wall and the y axis along the normal to it. The equations of particle motion in projections on these axes have the following form:

$$\frac{dU_p}{dt} = \beta (U_g - U_p) + g_x , \quad b = \frac{3}{2\pi} C\nu^{1/2} \frac{\rho_g}{\rho_p\delta} ,$$

$$\frac{dV_p}{dt} = - \beta V_p + b (U_g - U_p) \left(\frac{\partial U_g}{\partial Y} \right)^{1/2} + g_y . \tag{5}$$

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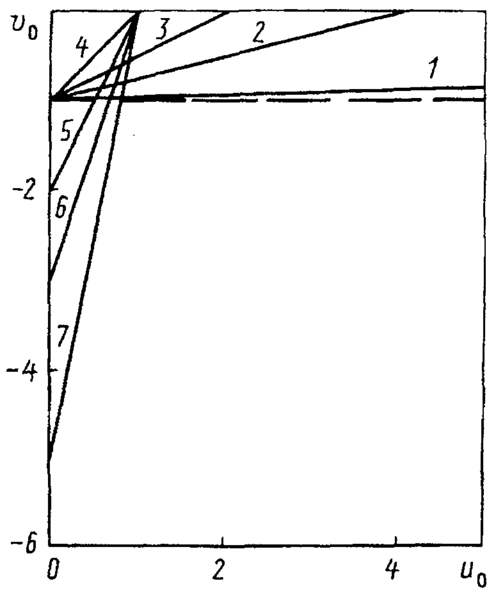

Fig. 1. Broken lines for determining the conditions for a particle to reach the wall at Stk = 1: 1) $\alpha_\zeta = 0.04$; 2) 0.25; 3) 0.5; 4) 1; 5) 4; 6) 9; 7) 25. Dashed straight line, $\alpha_\zeta \to 0$.

Let us now pass to dimensionless variables in system of Eqs. (5), having selected the following quantities as scales: U_∞ the gas velocity at the outer boundary of the layer and D the thickness of the Couette layer

$$\dot{u} = (y - u)/\mathrm{Stk} + \xi_x, \quad \dot{v} = -v/\mathrm{Stk} + \alpha(y - u) + \xi_y, \quad \xi_i = g_i D/U_\infty^2. \tag{6}$$

It is taken into account in Eqs. (6) that the dimensionless gas velocity in the layer $u_g = y$, $\alpha = \alpha_\zeta f$, $f = f_1 f_2$. Setting $f = 1$, it is possible to find an analytical solution of (6) [5]. We also assume that the values of ξ_i are rather small and can be neglected. Then, under the initial conditions $y_0 = 1$, $u_0 \geq 0$, and $v_0 < 0$ the second of Eqs. (6) has the following solution ($\alpha_\zeta \neq 1/\mathrm{Stk}^2$):

$$v = S_1 \exp(\lambda_1 \tau) + S_2 \exp(\lambda_2 \tau), \quad \lambda_1 = -1/\mathrm{Stk} - \sqrt{\alpha_\zeta}, \quad \lambda_2 = -1/\mathrm{Stk} - \sqrt{\alpha_\zeta}, \tag{7}$$

$$y = [\exp(\lambda_1 \tau) - 1] S_1/\lambda_1 + [\exp(\lambda_2 \tau) - 1] S_2/\lambda_2 + 1, \tag{8}$$

$$S_1 = 0.5\,[v_0 - (1 - u_0)\sqrt{\alpha_\zeta}\,], \quad S_2 = 0.5\,[v_0 + (1 - u_o)\sqrt{\alpha_\zeta}\,].$$

At $\alpha_\zeta = 1/\mathrm{Stk}^2$

$$y = [1 - \exp(-2\tau/\mathrm{Stk})]\, S_1\, \mathrm{Stk}/2 + S_2 \tau + 1. \tag{9}$$

In [5] the conditions were investigated under which a particle that penetrated through the upper boundary into the Couette layer could reach the wall. The results of [5], obtained for the case of $f = 1$ ($\alpha = \alpha_\zeta = \mathrm{const}$), will be given here in a form more convenient for application.

Let $\alpha_\zeta < 1/\mathrm{Stk}^2$, then with $\tau \to \infty$ the ordinate of the particle is

$$y_* = 1 - S_1/\lambda_1 - S_2/\lambda_2 = (\alpha_\zeta u_0 - v_0/\mathrm{Stk} - 1/\mathrm{Stk}^2)/(\alpha_\zeta - 1/\mathrm{Stk}^2). \tag{10}$$

The particle will reach the wall if $y_* \leq 0$; according to Eq. (10), this condition will be fulfilled at

$$v_0 \leq v_1 = \alpha_\zeta u_0\, \mathrm{Stk} - 1/\mathrm{Stk}. \tag{11}$$

The particle will not leave the layer if $0 < y_* < 1$; according to Eq. (10), this corresponds to $v_1 < v_0 < v_2 = \alpha_\zeta \mathrm{Stk}(u_0 - 1)$. When $v_0 \geq v_2$, the particle will leave the layer through the upper boundary.

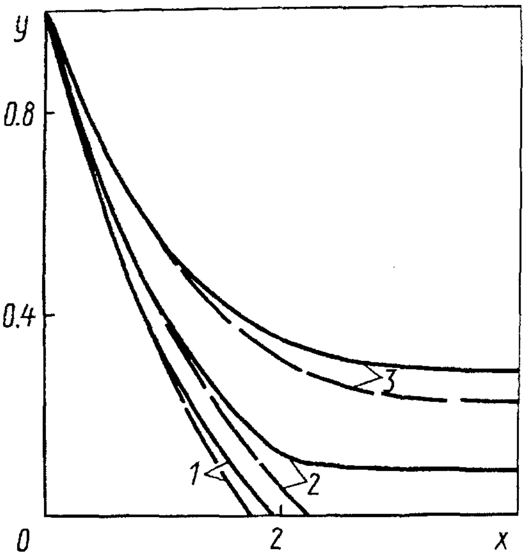

Fig. 2. Particle paths in a Couette layer at $\alpha_\zeta = 0.1$; $\tilde{\delta} = 0.05$; Stk = 1; $u_0 = 1$: 1) $v_0 = -1$; 2) $v_0 = -0.9$; 3) $v_0 = -0.7$. Dashed lines, calculation at $f = 1$.

Let $\alpha_\zeta \geq 1/\mathrm{Stk}^2$, then $\lambda_2 \geq 0$, and the particle will reach the wall when $S_2 < 0$; according to Eq. (10), this corresponds to the condition

$$v_0 < v_3 = \sqrt{\alpha_\zeta}\,(u_0 - 1)\,. \tag{12}$$

We verify now whether the particle will reach the wall when $\alpha_\zeta \geq 1/\mathrm{Stk}^2$ and $S_2 = 0$. Then $v_0 = v_3$, $y_* = 1 - S_1/\lambda_2$, whence $y_* \leq 0$ when

$$v_0 \leq -2/\mathrm{Stk} - \sqrt{\alpha_\zeta}\,(u_0 + 1)\,. \tag{13}$$

Substituting $v_0 = v_3$ into Eq. (13), we obtain $u_0 \leq -1/(\sqrt{\alpha_\zeta}\mathrm{Stk})$. Since we confine ourselves to the initial condition $u_0 \geq 0$, then the rigorous inequality (12) is the condition for reaching the wall when $\alpha_\zeta \geq 1/\mathrm{Stk}^2$.

Figure 1 presents the straight lines $v_0 = v_1(u_0)$ and $v_0 = v_3(v_0)$ plotted at different values of α_ζ. The condition for a particle to reach the wall is the position of the point $M(u_0, u_0)$ below the straight line plotted for the corresponding parameter α_ζ ($\alpha\zeta > 1/\mathrm{Stk}^2$, including the straight line). If the initial longitudinal velocity is larger than the value of u_0, which corresponds to the intersection of the straight line $v_0 = v_1(u_0)$ or $v_0 = v_3(u_0)$ with the axis $v_0 = 0$, then the condition for reaching the wall is $V_0 < 0$, i.e., the limiting lines are the broken lines in Fig. 1. It should be noted that with $\alpha_\zeta \to 0$ the slope of the limiting straight lines decreases, and they tend to occupy the position of the dashed straight line.

System of Eqs. (6) with account for the function $C(A, \eta)$ was solved numerically by the Runge-Kutta method. In solving it, we took into consideration that $\alpha_\zeta = 3.08\lambda/(\tilde{\delta}\mathrm{Re}_d)^{1/2}$, $\mathrm{Stk} = \tilde{\delta}^2\mathrm{Re}_d/(18\lambda)$, $\eta = y\mathrm{Re}_d^{1/2}$, and $A = v_r\mathrm{Re}_d^{1/2}$. Therefore, in calculations we should specify three determining parameters of the five constant quantities: Re_d, α_ζ, $\tilde{\delta}$, Stk, and λ.

Figure 2 presents the paths of particles at different initial transverse velocities v_0. It is evident that allowance for the corrections made by E. S. Asmolov (solid curves) leads to a change in the calculation results (dashed curves refer to calculations at $C = 6.46$). However, in the case of inertial precipitation of particles (curves 1) the differences are insignificant. Therefore, the results of calculations under the flow conditions of [6, 7] do not change much if the function $C(A, \eta)$ is taken into account. But curves 2 differ qualitatively, i.e., calculations without allowance for the corrections show that the particle reaches the wall, while those with corrections show that the particle remains in the layer. Obviously, E. S. Asmolov's corrections should be taken into account when determining the conditions for a particle to reach the wall.

Taking into account the function $C(A, \eta)$ at different values of α_ζ and $\tilde{\delta}$, we determined numerically the values of the initial transverse velocity v_0^* such that the particle could reach the wall when $v_0 < v_0^*$. It is established that in a wide range of parameters α_ζ and $\tilde{\delta}$ the value of v_0^* differs from $-1/\mathrm{Stk}$ only by a small positive value of

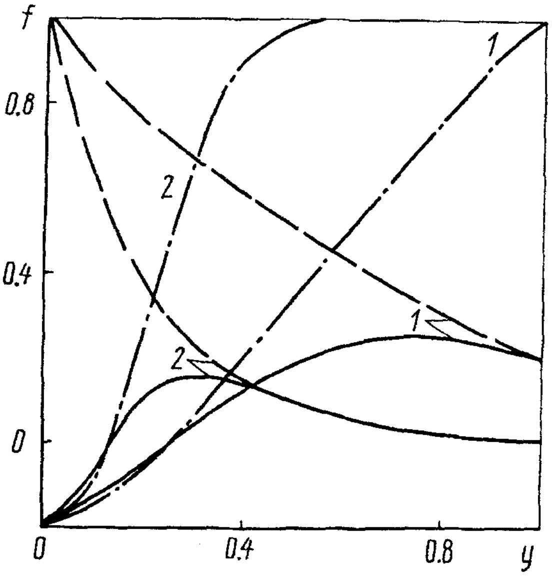

Fig. 3. Variation of E. S. Asmolov's corrections across a Couette layer at $\tilde{\delta}$ = 0.05; Stk = 1; $u_0 = 1$; $v_0 = -1$: 1) $\alpha_\zeta = 0.02$; 2) 0.04. Solid lines refer to f, dashed lines refer to f_1, dashed-dotted lines refer to f_2.

ε, i.e., $v_0^* = -1/\mathrm{Stk}+\varepsilon$. In order to find the reason for this, we consider the change in the functions f_1, f_2, f across the layer.

Figure 3 shows the change in f_1, f_2, f at two values of α_ζ. Far from the wall, the value of f_1 is equal to unity; it decreases in approaching the wall and then changes sign. Conversely, the value of f_2 increases to unity near the wall. This is a consequence of the decrease in v_r near the wall and, consequently, in A. Having a maximum, the function f remains much smaller than unity in absolute value. Moreover, as α_ζ increases, the value of the maximum decreases, so that the quantity $\alpha = f\alpha_\zeta$ is always of the order of 10^{-3}. This corresponds to a limiting line that differs little from the dashed line in Fig. 1; therefore, the value of $v_0^*\mathrm{Stk}$ is close to unity.

NOTATION

$x = X/D$, $y = Y/D$, dimensionless longitudinal and transverse coordinates; $u = U_p/U_\infty$, $v = V_p/U_\infty$, dimensionless projections of particle velocity on the longitudinal and transverse axes; $\tau = tU_\infty/D$, dimensionless time; $\mathrm{Stk} = U_\infty\delta^2/(18\nu\lambda D)$, Stokes number, $\lambda = \rho_g/\rho_p$, ν, coefficient of the gas kinematic viscosity, δ, particle diameter; $\tilde{\delta} = \delta/D$; ρ_g, ρ_p, densities of the gas and particle material; $\dot{u} = du/d\tau$, $\dot{v} = dv/d\tau$, P_s, Saffman's force; C, coefficient in the formula for Saffman's force; $\eta = y\mathrm{Re}_d^{1/2}$; $A = v_r\mathrm{Re}_d^{1/2}$; $\alpha_\zeta = 3.08\lambda/(\tilde{\delta}\mathrm{Re}_d)^{1/2}$; $\mathrm{Re}_v = \delta V_r/\nu$; $\mathrm{Re}_k = (\delta^2/\nu)\partial U_g/\partial Y$; $A \equiv \mathrm{Re}_v/\mathrm{Re}_k^{1/2}$; $\mathrm{Re}_d = U_\infty D/\nu$; $V_r = ((U_g-U_p)^2+V_p^2)^{1/2}$. Indices: g refers to gas parameters, p refers to the parameters of particles, 0, at the time moment $t = 0$; S, Saffman's force; k, Reynolds number based on the velocity gradient, v, based on velocity; r, relative velocity; x, projection on the x axis.

REFERENCES

1. P. G. Saffman, J. Fluid Mech., 22, No. 2, 385-400 (1965).
2. E. S. Asmolov, Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 6, 91-96 (1990).
3. E. S. Asmolov, Izv. RAN, Mekh. Zhidk. Gaza, No. 1, 66-73 (1992).
4. A. A. Shrayber, L. B. Gavin, V. A. Naumov, and V. P. Yatsenko, Turbulent Gas Suspension Flows [in Russian], Kiev (1987).
5. M. A. Brich, Heat and Mass Transfer: Results and Perspectives [in Russian], Minsk (1985).
6. V. A. Naumov, Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 6. 171-173 (1988).
7. V. A. Naumov, Izv. RAN, Mekh. Zhidk. Gaza, No. 2, 186-187 (1992).